Non-Newtonian Heat Transfer in the Thermal Entrance Region of Uniformly Heated, Horizontal Pipes

Experimental data were obtained for laminar forced flow in the thermal entry region of a uniformly heated pipe with fluids exhibiting a range of non-Newtonian behavior. The Graetz number range was between 240 and 38,000. Contributions to heat transfer made by non-Newtonian fluid behavior, temperature-dependent fluid properties, viscous heating, and secondary flow due to buoyancy were evaluated.

Results indicate that heat transfer is controlled by local wall shear rates. Local wall shear is affected much more by temperature variation of fluid rheological properties than by the degree of pseudoplasticity. Viscous heating was not detected in any of the tests. Natural convection effects were apparent in all cases; however, the heat transfer rates showed contributions due to buoyancy only at the lowest flow rates with low viscosity fluids.

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SCOPE

Considerable investigative effort has been expended in the past, both experimental and analytical, on the subject of heat transfer to non-Newtonian fluids in horizontal, isothermal pipes. This effort was motivated by the popular use of isothermal boundaries in various processing industries. However, the advantages of the uniformly heated or cooled (constant heat flux) boundary cannot be overlooked. More energy can be delivered to (or extracted from) the fluid per unit area for a given wall temperature constraint under conditions of uniform wall heat flux.

In the case of heating, an additional heat transfer contribution is available from natural convection. Although natural convection effects have been reported with isothermal pipes, only limited increase is possible in this case. The maximum contribution to heat transfer rates from natural convection has been shown to occur somewhat downstream of the entrance where, in the case of an isothermal boundary, the bulk temperature asymptotically approaches the wall temperature value. In the limit, as the temperature difference vanishes, the buoyant mechanism is shut off. In the case of uniform heating, however, a temperature difference always exists between the wall and the bulk flow. Moreover, this difference increases to a maximum as the flow progresses downstream, thereby implementing and feeding the buoyant mechanism which drives the secondary flow.

Recent experimental studies in Newtonian fluids including air, water, and ethylene glycol have confirmed that increases in the rate of heat transfer of 3 to 4 times that indicated by a constant property analysis are possible in the region of full thermal development. Theoretical work predicts that large increases are also possible in higher Prandtl number fluids.

Uniform heating (or cooling) of non-Newtonian fluids in laminar flow in horizontal pipes has been studied analytically for several rheological models, some of which were temperature dependent. However, no significant experimental studies have been undertaken to address this problem.

The present study was initiated specifically to determine experimentally the rate of heat transfer to pseudoplastic (shear thinning) fluids in laminar flow in the thermal entrance region of uniformly heated, horizontal pipes. A secondary objective was to document the presence of secondary flows due to buoyancy and determine their effect on the rate of heat transfer.

To achieve these aims, the polymers sodium carboxymethylcellulose (CMC) and polyethylene oxide, coagulant grade (polyox) were used in various concentrations in aqueous solutions to obtain a range of pseudoplastic behavior. Fluid rheology was determined using a Haake rotational viscometer. A unique method was employed to apply the viscometric results to the conditions in the pipe flow.

CONCLUSIONS AND SIGNIFICANCE

Results were obtained for Graetz numbers in the range 240 to 38,000. It was found that temperature-dependent viscous properties affected the local rate of heat transfer to a much greater degree than non-Newtonian effects. Two pairs of runs, shown in Figures 4 and 5, demonstrate that a 5.4% solution of CMC is inferior to a 2.4% solution of polyox with respect to heat transfer at low flow rates. At higher flow rates the opposite becomes true. The polyox fluid exhibited greater pseudoplasticity throughout the range of flow rates; however, the CMC solution demonstrated greater viscous dependence on temperature than did the polyox fluid, especially at higher flow rates.

Explicit natural convection effects on the rate of heat transfer were detected in two of the 26 runs. They occurred in the two least viscous fluids tested in the large diameter test section at the lowest flow rates. There were

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insufficient data to obtain a criterion for the onset of significant natural convection effects; however, evidence of secondary flow due to buoyancy was obtained in runs other than these two. Some secondary flow patterns developed in even the most viscous fluids run in the largest test section. The intensity of these patterns decreased with increasing flow rate, increased with the less viscous fluids, and increased as the flow moved downstream. In addition, with some runs, including all those at low flow rates in the large test section, a temperature difference between the top and the bottom of the pipe wall was detected downstream of the entrance. This difference became greater as the flow approached the exit.

No effect from viscous heating was found in any of the fluids tested. The data, exclusive of the two runs exhibiting convection effects, were correlated within ± 10% by the expression

$$Nu_x = 1.85\, Gz_x^{1/3 - 0.03/\delta_x}$$

It was concluded that the local wall shear rate controls the rate of heat transfer and that the shear rate is more profoundly affected by temperature variation of viscous properties than by non-Newtonian behavior. Further, it was concluded that natural convection affects the rate of heat transfer in non-Newtonian fluids under certain favorable viscosity and flow conditions and that secondary flow due to buoyancy can exist in the most viscous fluids even though an explicit effect on the rate of heat transfer is not evident.

Heat transfer in the thermal entry region of laminar, non-Newtonian, horizontal pipe flows is controlled by contributions from several sources. Potential supplements to the basic conduction mechanism at the boundary are the temperature effects on transport properties (particularly viscosity), the shear rate dependence of viscosity, the development of secondary flows due to buoyancy, and normal stress phenomena (in viscoelastic fluids). It has been difficult to isolate the degree of contribution from each source in the laboratory. Most recent investigations into secondary flows, for example, have been numerical solutions to the coupled equations of continuity, momentum, and energy for Newtonian fluids with temperature-independent properties.

In the case of non-Newtonian fluids, both temperature dependence and shear rate dependence of viscous properties promote changes in the velocity profile. The extent to which each controls is masked by the other. As a consequence, empirical expressions have been based on temperature-independent property solutions obtained from analysis. Modifying parameters such as K_b/K_w or η_b/η_w have been introduced to account for temperature dependence and the effects from natural convection added as a separate term.

The problem of the presence of normal stresses in laminar, viscoelastic flows has not been addressed. Normally, laminar pipe flows are viscometric and viscoelastic effects do not occur; however, in flows where secondary patterns have developed, this is not the case.

Much is known about forced convection heat transfer between isothermal pipes and non-Newtonian fluids. The correlations from investigations with isothermal pipes have been formulated in terms of average values of the pertinent parameters. Unfortunately, these correlations provide no useful information to the designer of a uniformly heated or cooled system where local values are of interest. A number of analytical studies have been made of the problem; however, little significant experimental data have been obtained. The present investigation was initiated to provide that experimental information.

HISTORY

The extension of the original solution by Graetz (1885) for a constant property Newtonian flow in an isothermal pipe to that of uniform heating or cooling at the boundary was first made by Sellars, Tribus, and Klein (1956). Siegel, Sparrow, and Hallman (1958) obtained an improved solution. The asymptotic relationship

$$Nu_x = 1.411\, Gz_x^{1/3} \tag{1}$$

is a quite accurate representation of that solution in most of the thermal entry region ($Gz_x > 50\pi$).

Yang (1962) showed, by an integral solution, that uniform heating or cooling has a potentially greater effect on viscosity (and thus heat transfer) than does an isothermal boundary. Test (1968), modeling SAE 60 oil, showed as much as 50% deviation from the constant property solution to be possible.

The investigation of secondary flows in horizontal, uniformly heated pipes has thus far been restricted to Newtonian fluids. Experimental studies in the fully developed region have been conducted by Bergles and Simonds (1971) for water, Mori et al. (1966) for air, and Siegwarth and Hanratty (1970) for ethylene glycol. Mori et al. provided visual evidence of a pair of vortices with centers located below the horizontal center line of the cross section and symmetrically positioned on either side of the vertical centerline. Bergles and Simonds used a glass tube to limit circumferential conduction at the boundary. At the other extreme, Siegwarth and Hanratty used an aluminum pipe with walls 2.54 cm thick to eliminate circumferential temperature gradients. A numerical study by Newell and Bergles (1970) which modeled these two extreme boundary conditions showed a definite heat transfer advantage in using the thick, high conductivity wall.

Other analytical studies in the fully developed region have been conducted by Morton (1959), Mori and Futagami (1967), Faris and Viskanta (1969), Siegwarth et al. (1969), and Hwang and Cheng (1970). Hwang and Cheng used a boundary vorticity method for the purpose of bridging the gap between the perturbation techniques used by Morton and by Faris and Viskanta, which are confined to the lower range of Gr^*, and the boundary layer-core region techniques used by Mori and Futagami and by Siegwarth et al. which are best suited to the higher range of Gr^*. Del Casal and Gill (1962) extended Morton's solution to cases of smaller Re.

McComas and Eckert (1966) using air, Shannon and Depew (1968, 1969) using water and ethylene glycol, and Petukhov and Polyakov (1967) using water have extended studies into the thermal entrance region. Petukhov and Polyakov have suggested as transition criteria for the onset of significant secondary flow the following:

$$Ra^*_{tr} = 6370\, Gz_x, \quad Gz_x > 460 \tag{2}$$

$$Ra^*_{tr} = 18000 + 83\, Gz_x^{1.7}, \quad Gz_x < 460 \tag{3}$$

Their suggested correlation for heat transfer was

$$Nu_x = Nu_{x0}\left[1 + \left(\frac{Ra^*}{Ra^*_{tr}}\right)^4\right]^{0.045} \qquad (4)$$

Cheng et al. (1972) obtained an analytical solution for the thermal entrance region in rectangular ducts. This analysis specified large Prandtl numbers (>10) with only the density allowed to vary with temperature. In the case of a square duct, temperature profiles first showed vertical distortion due to secondary flow far upstream of the fully developed region.

The problem of uniform heating of non-Newtonian fluids in the fully developed region has been addressed analytically by Lyon (1951) for arbitrary temperature-independent rheology. Sestak and Charles (1968), using the power law model, extended Lyon's solution to include viscous dissipation.

Bird (1955) obtained a series solution for temperature-independent power law fluids. More importantly, he solved for the asymptotic relationships appropriate to the two thermal regimes. In the thermal entrance region he obtained

$$Nu_x = 1.411\left(\frac{3n+1}{4n}\right)^{1/3} Gz_x^{1/3} \qquad (5)$$

which is less than 1% different than the complete series solution if $\frac{4}{\pi}\left(\frac{3n+1}{4n}\right) Gz_x > 100$. It is convenient to note here that δ, the shear rate ratio, $\dot{\gamma}/(8V/D)$, first used by Pigford (1955) is

$$\delta = \frac{3n+1}{4n} \qquad (6)$$

for power law fluids. Hence, the Bird solution may also be expressed as

$$Nu_x = 1.411\,\delta^{1/3}\,Gz_x^{1/3} \qquad (7)$$

Matsushisa and Bird (1965) used this concept to modify the solution to fit the Ellis model parameters.

Gill (1962) extended the complete series solution to include viscous dissipation, and Michiyoshi and Matsumoto (1964) added uniform heat generation. Other temperature-independent models used were the Ellis model by Mitsuishi and Mayatake (1967), Bingham plastic model by Michiyoshi et al. (1962, 1963), and Prandtl-Eyring model by Schenk and Van Laar (1958). The latter case was for uniform cooling. All of these series solutions include eigenvalues which must be evaluated in terms of the rheological model parameters. As a result, application is tedious, and their usefulness is diminished. McKillop (1964) avoided this shortcoming by solving the equations of continuity, momentum, and energy numerically. He chose the power law model.

Employing a temperature-dependent power law, Mizushina et al. (1967) modified Bird's result with the term $(K_b/K_w)^{0.1/n^{0.7}}$. The result was compared to data taken on glycerol (Newtonian) and aqueous CMC for $10 < Gz_x < 300$, near the end of the thermal entrance region. Significant scatter suggests that buoyant effects were present but not accounted for.

Mitsuishi and Miyatake (1968, 1969), also using the temperature-dependent power law, derived Equation (7) specifying δ as a function of the heat flux and the rheological parameters. Experimental data were taken in a vertical pipe for aqueous CMC in the region, $25 < Gz_x < 1500$.

Results for a temperature-dependent power law

$$\tau = K\,(\dot{\gamma}\,e^{\Delta H/RT_0})^n \qquad (8)$$

were also obtained by Cochrane (1969, 1972) using an analysis similar to that of McKillop. In the first investigation the boundary was heated, and in the second, cooled. For $\phi = 2$, $n = 1$, $Pr_0 = 1000$, and $\Delta H/RT_0 = 5$, he found that the increase in Nu_x was 5% at $Gz_x = \pi/4 \times 10^5$ and 14% at $Gz_x = \pi/4 \times 10^2$ compared to the constant property solution [Equation (1)]. At $\Delta H/RT_0 = 10$ and all other conditions the same, the increases were 7 and 27%, respectively. In the case of cooling, with $\phi = -0.25$, the decrease in Nu_x was almost twice as great as the increase for $\phi = 0.25$ (heating) compared to the constant property solution.

Bader et al. (1970) also presented a numerical solution using a temperature-dependent power law. Instead of the usual fully developed velocity profile at entrance, a uniform flow was specified. Results were compared with data taken on aqueous hydroxyethylcellulose (HEC). The data are difficult to interpret, and Bader's thesis has not been completed.

Khabakhpasheva et al. (1970) and Kutateladze et al. (1970) report what are apparently the results from the same data on the flow of aqueous polyacrilamide. The data are presented in terms of a rheological parameter which has not been used in Western literature. In addition, the local Nusselt numbers have been "reduced to quasi-isothermal conditions by extrapolating to zero values of the heat flux." These factors make comparison of these data with other results virtually impossible.

Etchart (1971) obtained a numerical solution for temperature-dependent Powell-Eyring fluids using rheological data from the literature. Forrest and Wilkinson (1973) solved numerically for the cases of heating and cooling of temperature-dependent nonlinear plastics.

EXPERIMENTAL APPARATUS

The test apparatus used in this investigation is shown in Figure 1. The Moyno, 2L4 pump was driven by a 2 HP DC motor. Static mixers upstream and downstream of the test sections provided mixing needed prior to bulk temperature measurement. Thermocouple probes were located downstream of each mixer.

Piping upstream and downstream of the test sections was PVC or CPVC, thereby limiting heat conduction losses. Entrance sections were 152.5 cm in length and flanged to the test sections. The smaller section was constructed from ½ in., schedule 80, PVC pipe (1.385-cm I.D.). The larger from 1 in., schedule 40, PVC pipe (2.664-cm I.D.). Test sections were 305 cm in length, constructed from hard drawn copper tubing. Cross section dimensions were 1.588-cm O.D. and 1.384 I.D. for the small test section. Corresponding dimensions for the larger test section were 2.858 cm and 2.680 cm.

The test sections were heated using 0.318 cm wide by 0.127 mm thick nichrome ribbons arranged longitudinally along the outer surface. The ribbon was rated at 3.05 ohms/m by the

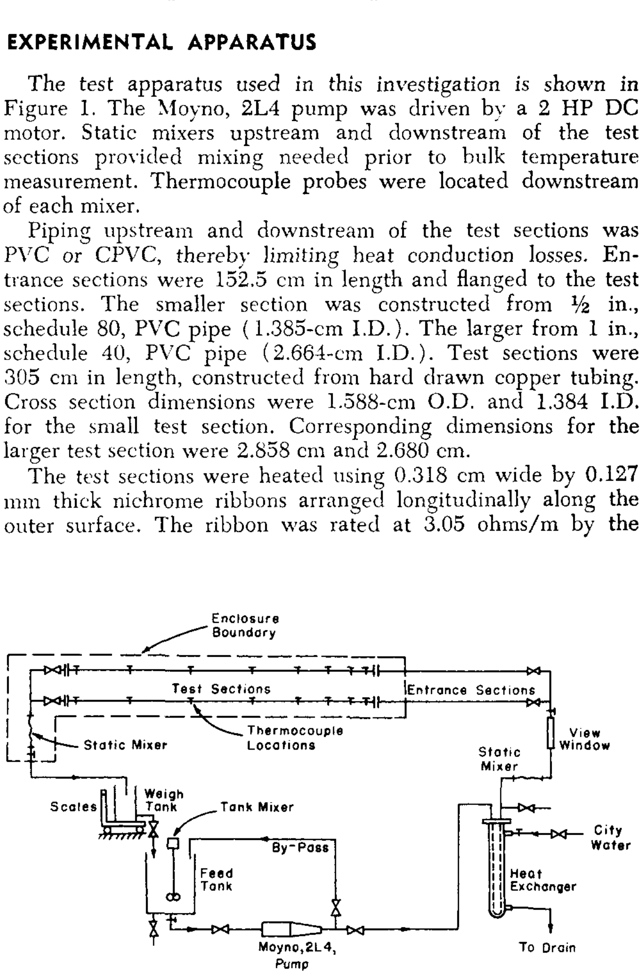

Fig. 1. Schematic diagram of test apparatus.

manufacturer. The wiring was arranged so that the total area of the heating elements was 102% of the inside surface area of the small test section and 98.1% of the inside area of the large test section. Overall heater resistance was 7.92 ohms for the small test section and 6.50 ohms for the larger test section. Insulation between the elements and the test section walls consisted of a 0.0762-mm layer of mylar-backed tape (adjacent to the wall) followed by a 0.178-mm layer of fiberglass-backed tape. The heaters were supplied by a Sorensen model DCR 300-35A regulated DC power supply rated at 0-300 volts and 1-35 amps.

Copper-constantan thermocouples were soldered to the test section walls, top and bottom, at positions 7.62, 22.86, 45.72, 76.2, 121.92, 182.88, 243.84, and 297.18 cm from the entrance. They were wired in a precision circuit suggested by Benedict (1969). They were calibrated on the test sections prior to initiation of the test program.

The test sections were installed in a 61-cm wide by 46-cm deep wood enclosure which was filled with vermiculite insulation. All test section supports were constructed of wood or plastic materials and the contact areas were minimized to limit heat losses. A complete description of the test apparatus and an account of the construction are provided by Bassett (1975).

Fluid rheology was determined using a Haake Rotovisco rotational viscometer. Sample temperature was regulated by a Haake, Model FE circulator which was capable of controlling the circulating fluid within ±0.02 C. A complete description of the viscometer and an evaluation of its capabilities is provided by Van Wazer et al. (1963).

EXPERIMENTAL PROCEDURE

Two aqueous CMC solutions and 4 aqueous polyox solutions were used during the test program. The CMC solutions were 3.0% and 5.4% concentrations (nominal). The polyox solutions were 1.0%, 1.6%, 2.4%, and 2.4% (degraded). The test fluids were circulated through the test sections until it was apparent that their rheology would remain stable for at least 8 hours of test time. In meeting this requirement, more polymer than usual was needed in order to offset degradation and maintain the desired degree of pseudoplasticity.

Data on all but the 2.4% (degraded) polyox solution were obtained in each of the test sections at mass flow rates in the vicinity of 100 g/s and 275 g/s. During these runs, power input to the heaters was adjusted such that the maximum wall temperature was approximately 83 C. Inlet temperature was adjusted to room temperature, 21 to 23 C, within 0.1 C. Data on the 2.4% (degraded) polyox solution were taken at flow rates in the vicinity of 275 g/s and 415 g/s. At the lower flow rate, a power input of approximately 3000 watts was applied to each test section. At the higher flow rate, runs were made with power input of approximately 6000 and 6900 watts to each test section.

Between each heat transfer run, a viscometer run was made on the test fluid. These viscometer runs, with the sample temperature controlled at 20 C, provided a check on the stability of rheological behavior. At the end of the 4 heat transfer runs (6 for the degraded polyox solution), a complete set of viscometer runs was made. These runs were conducted with the sample temperature controlled at 20, 30, 40, 50, 60, 70, and 83 C.

PROCESSING OF EXPERIMENTAL DATA

Wall temperatures were corrected to account for small gaps in the heating at the thermocouple locations. Details of these corrections are discussed by Bassett (1975). The properties of the test fluids, other than the viscous properties, were taken to be the same as those of pure water. Heat capacity and density were found by Christiansen and Craig (1962) to be the same as those of the pure solvent for concentrations of CMC similar to those employed in this investigation. They also found that thermal conductivity was within 1 to 3% of that of the pure solvent. Oliver and Jenson (1964) reported that the thermal conductivity of aqueous solutions of polyox were no more than 5% less than those of pure water.

Local bulk temperatures were determined from a simple heat balance

$$T_b = T_0 + \pi D x q'' / \dot{m} C_p \tag{9}$$

Local Nusselt numbers using thermal conductivity evaluated at the local wall temperature were chosen for correlation. This choice is supported by the classic definition of the Nusselt number in terms of its physical significance; that is, as the ratio of the wall temperature gradient to the temperature gradient across the flow. Properties in the local Graetz number were evaluated at the local bulk temperature.

The viscometric data were processed in such a way as to avoid embracing a particular rheological model. The measured quantities were related using the fundamental equations for concentric cylinder viscometers (see Van Wazer, p. 51):

$$\tau r^2 = \text{constant} \tag{10}$$

$$\Omega = -\tfrac{1}{2}\int_{\tau_1}^{\tau_2} \dot{\gamma}\,\frac{d\tau}{\tau} \tag{11}$$

Conditions on these equations are one-dimensional, steady, laminar, incompressible, isothermal flow. Modifying a method suggested by Huang (1971), $\dot{\gamma}$ was determined explicitly from Equation (11). It was assumed

$$\dot{\gamma} = C_1\tau + C_2\tau^2 + C_3\tau^3 + \ldots \tag{12}$$

where C_i are unknown constants to be determined. Substitution of Equation (12) into Equation (11) yields

$$\frac{2\Omega}{\tau_1} = B_0 + B_1\tau_1 + B_2\tau_1^2 + \ldots \tag{13}$$

A least squares fit of the data at a particular temperature provided B_i and from them, C_i.

In order that the constitutive equations from Equation (12) be applied to the pipe flow, a form of the Mooney-Rabinowitch equation was used (see Skelland, 1967, p. 70):

$$\frac{8Q}{\pi D^3} = \frac{1}{\tau_w^3}\int_0^{\tau_w} \dot{\gamma}\,\tau^2 d\tau \tag{14}$$

Conditions on its use are steady, laminar, fully developed flow. These conditions were met just prior to entry to the heated section. A numerical integration of Equation (14) was accomplished using logarithmic interpolation between the constitutive equations (which were obtained for discrete temperatures).

Results from the analysis by Etchart (1971) reveal that the axial pressure gradient remains essentially constant over the entire thermal entrance region. (Some deviation from linearity does occur at the lower values of Gz_x, becoming more pronounced as the degree of pseudoplasticity increases.) A uniform pressure gradient suggests a constant value of wall shear stress in the absence of significant changes in momentum flux. On this basis, the wall shear stress was assumed not to vary from entrance to exit.

The constitutive equations were extended using the power law. Then, using extrapolation and the value of the wall shear stress previously obtained, the local wall shear rates were estimated.

An apparent viscosity at the conditions of the bulk flow η_b was obtained from a mixing cup analogy. The relationship, first suggested by Charm and Merrill (1959), is

$$\eta_b = \frac{\int_A u\eta\, dA}{\int_A u\, dA} = \frac{2\pi}{Q}\int_0^{D/2} u\eta r\, dr \tag{15}$$

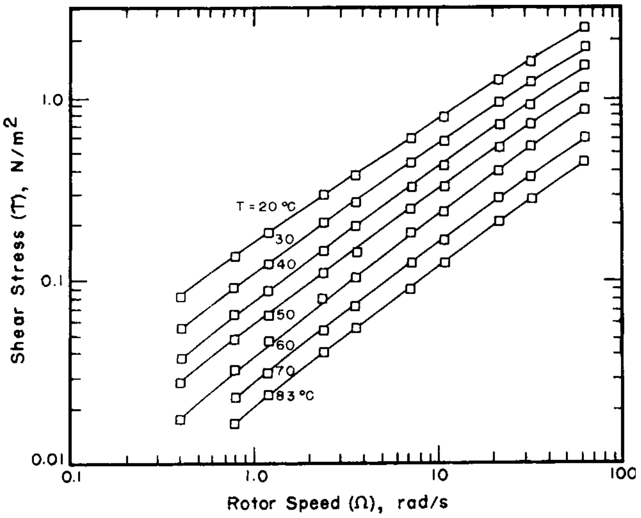

Fig. 2. Viscometry results for 5.4% CMC solution.

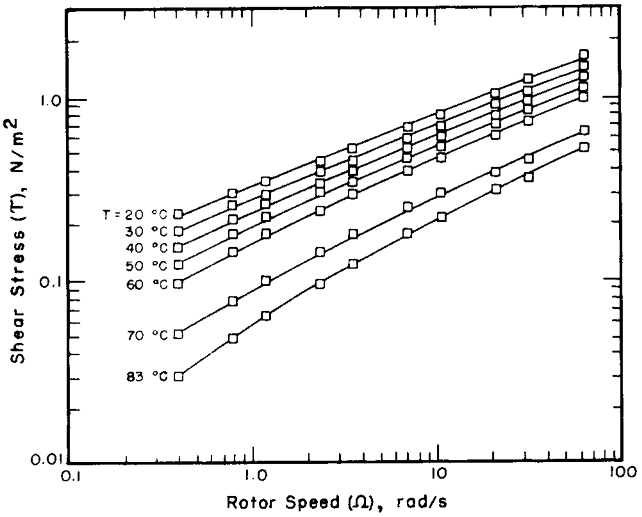

Fig. 3. Viscometry results for 2.4% polyox solution.

For this evaluation, the whole cross section was treated as being at the bulk temperature. For such an isothermal condition,

$$\frac{\tau}{r} = \text{constant} \qquad (16)$$

Using Equation (16), the variable of integration in Equation (15) was changed and the equation integrated numerically. Values of u were calculated by integrating $\dot{\gamma}$.

The parameters Pr, Re, Gr^*, Ra^*, δ, and η_b/η_w were generated at local conditions (both wall and bulk conditions, if appropriate). Entrance values of Pr, Re, Br, δ, and η_b/η_w were also computed. Three generalized models were tested using the generated parameters as the independent variables.

Model 1: $$Nu_x = C_1 X_1^{m_1} X_2^{m_2} X_3^{m_3} \ldots \qquad (17)$$

Model 2: $$Nu_x = C_1 Gz_x^{1/3 + C_2X_2 + C_3X_3 + \ldots} \qquad (18)$$

Model 3: $$\frac{Nu_x}{1.411\,\delta_0^{1/3}\,Gz_x^{1/3}} = X_1^{m_1} X_2^{m_2} X_3^{m_3} \ldots \qquad (19)$$

The parameters were added to the models, their significance tested statistically using an F-test, and dropped if their contribution to the model was not convincing. Evaluation of the model constants was accomplished by least squares fit of the data to the logarithmic form of the model equations.

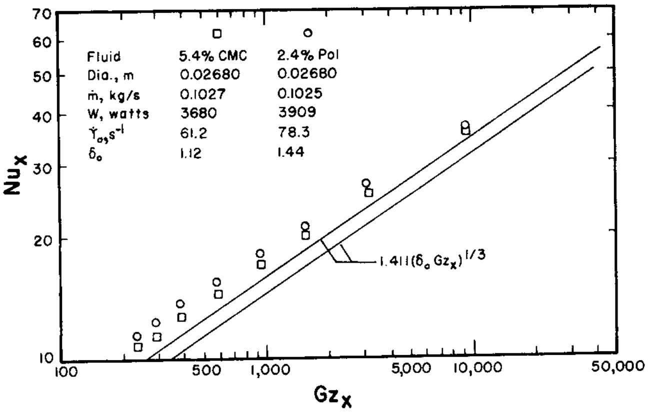

Fig. 4. Heat transfer results for 5.4% CMC and 2.4% polyox at low mass flow rates in large test section. Solid lines represent temperature independent solutions.

RESULTS OF TEST

Viscometer data for the 5.4% CMC and the 2.4% polyox solutions are shown in Figures 2 and 3. Since $\dot{\gamma}$ is directly related to the rotor speed Ω, the trend of the data are similar to a τ vs. $\dot{\gamma}$ plot. Note that the effect of temperature on the viscous behavior of the 5.4% CMC solution is more pronounced than is the case with the 2.4% polyox. The difference in effect is especially evident at higher shear rates.

The relative viscous behavior of these solutions at different temperatures and shear rates can be related directly to the heat transfer data obtained with them. Results of individual runs of the 5.4% CMC and the 2.4% polyox solutions under comparable conditions are shown in Figures 4 and 5. The lines below the data represent the temperature-independent property solution, Equation (7), using the entrance value of δ. The difference between the data and the corresponding line is due to the effect of temperature on the rheology. The polyox fluid, displaying a greater degree of pseudoplastic behavior, corresponds to the top line. In both sets of runs, the difference between the temperature-independent property line and the data is greater for the CMC runs than for the polyox. At the low entrance value of shear rate for the runs in Figure 4, the polyox data are above the CMC data, probably due to the more pronounced pseudoplastic behavior of the former fluid. However, as the shear rate at entry is increased, the difference in temperature-dependent rheology offsets the difference in pseudoplastic behavior. Finally, as shown in Figure 5, at

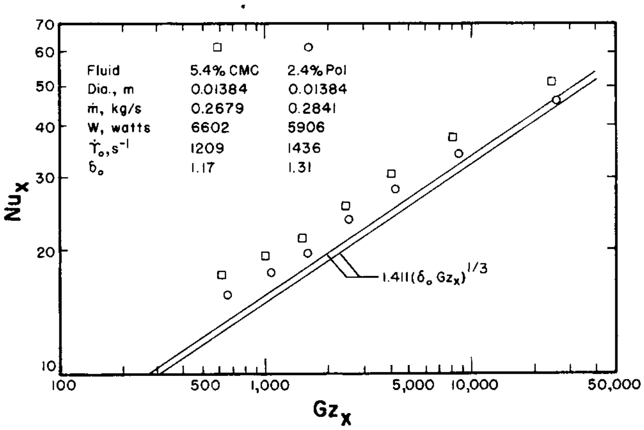

Fig. 5. Heat transfer results for 5.4% CMC and 2.4% polyox at moderate mass flow rates in small test section. Solid lines represent temperature independent solutions.

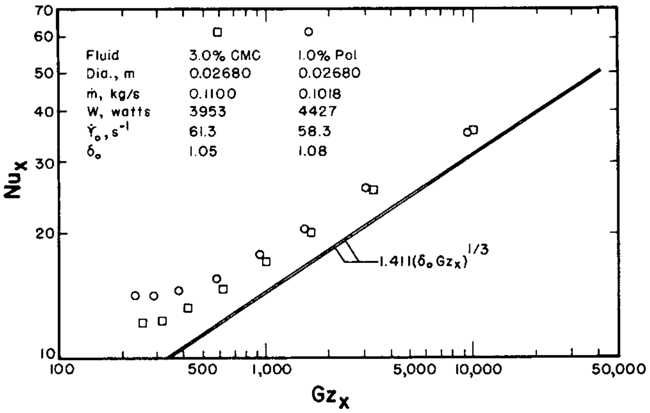

Fig. 6. Heat transfer results for 3.0% CMC and 1.0% polyox at low flow rates in large test section. Solid lines represent temperature independent solutions.

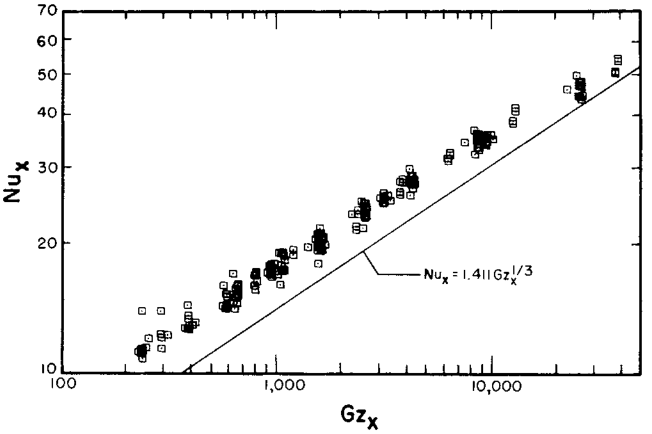

Fig. 7. Heat transfer results for all data. Solid line represents temperature independent solution.

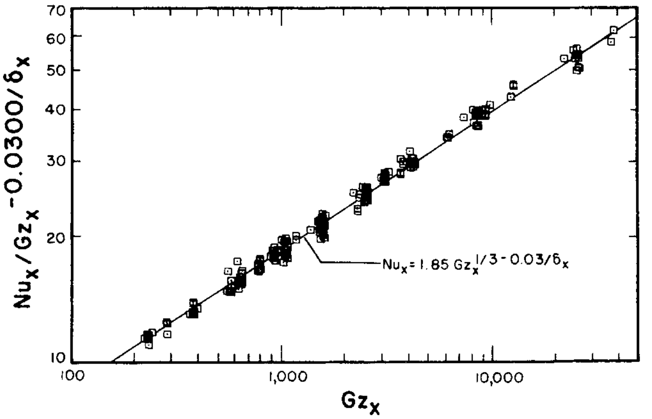

Fig. 8. Best correlation of data which did not show an obvious effect from natural convection.

a high entrance value of shear rate, the rate of heat transfer to the CMC solution is greater than to the polyox solution. This trend was observed consistently for all 26 test runs.

Figure 6 shows results from the only runs in which it was apparent that natural convection had an effect on the rate of heat transfer. The runs were made in the larger test section at the lowest flow rates. The upper set of data are for the 1.0% polyox solution, the least viscous of the test fluids used. The lower set of data are for the 3.0% CMC solution. The familiar contour at the downstream end of the plot is evident, indicating the onset of nearly full thermal development. An insufficient amount of data were obtained to document a criterion for the onset of significant secondary flow. Roughly, the point of departure from linearity (on a log-log plot) appears to take place at $Gz_x = 600$ for the 1% polyox fluid. At this point $Ra_w^* = 3.47 \times 10^7$ and $Ra_b^* = 3.02 \times 10^6$ were calculated. For the CMC fluid, the corresponding point appears to be $Gz_x = 400$, at which $Ra_w^* = 2.00 \times 10^7$ and $Ra_b^* = 1.83 \times 10^6$ were calculated. The transition criteria of Petukhov and Polakhov (1967) predict $Ra^* = 3.82 \times 10^6$ at $Gz_x = 600$ and $Ra^* = 2.22 \times 10^6$ at $Gz_x = 400$.

Evidence of the existence of secondary flow was found in other runs also. During some of the tests at moderate flow rates (about 0.28 kg/s) in the large test section and some of the tests at low flow rates (about 0.1 kg/s) in the small test section, oscillations were apparent in the readings of some of the top thermocouples. These oscillations occurred in every run made in the larger test section at the low flow rate. Never was an oscillation noted for a bottom thermocouple. In general, the less viscous the fluid, the further upstream the oscillations first appeared. The intensity of the oscillations was more pronounced in the less viscous fluids, increased as the flow progressed downstream, and decreased with increasing flow rate.

Other evidence of secondary flow was obtained from wall temperature data. Temperature differences between the top and bottom thermocouple readings were evident in runs with even the most viscous fluids. For example, for the run shown in Figure 4 with the 5.4% CMC solution, the difference in the top and bottom readings was 2.7°C at the last station downstream. Since considerable circumferential conduction in the tube wall at such high temperature gradients was possible, a reliable transition criteria could not be based on these observations.

The heat transfer data for the entire investigation (195 points) are shown in Figure 7. These data are contained in an envelope approximately +18%, −12% from a least squares line. Correlation of the data, excluding the two runs where natural convection was explicit in the results, was best accomplished by the equation

$$Nu_x = 1.85\, Gz_x^{1/3-0.03/\delta_x} \qquad (20)$$

None of the other parameters was found to be significant when added to the model after δ_x was included. The results of this correlation are shown in Figure 8. The spread in the data was reduced to ± 10% with a mean error of 3.57%.

Finally, it is probable that viscous heating had very little or no effect on the heat transfer results. When the most viscous fluid used (2.4% polyox) was run through the small test section at the highest flow rate (without applying heat), no difference in wall temperature could be observed from entrance to exit. The maximum Br_0 in this investigation was 4.22×10^{-3}.

CONCLUSIONS

This experimental study has provided comprehensive data for heat transfer to pseudoplastic fluids in the thermal entrance region of uniformly heated, horizontal pipes. From the heat transfer results, it can be concluded that:

1. The local rate of heat transfer with temperature-dependent fluid properties is greater than that for no temperature dependence at Graetz numbers as high as 38,000. Further, the difference increases as the flow progresses downstream (Graetz number decreases).
2. Normally, temperature-dependence of fluid rheology has more effect on the rate of heat transfer than does the degree of pseudoplasticity.

3. Secondary flows due to buoyancy can have a substantial effect on the rate of heat transfer far upstream of the usual onset of full thermal development. The point at which this effect becomes evident moves upstream for less viscous fluids.

4. Secondary flow patterns which have no obvious effect on the rate of heat transfer can exist far upstream of full thermal development in fluids with Prandtl numbers as high as 20,100.

5. The relative value of the local wall shear rate controls the magnitude of the rate of heat transfer. The relationship, Equation (20), determines the rate of heat transfer within ±10%, with a mean error of 3.57% for flows without significant natural convection effects.

6. Viscous heating made no significant contribution to the rate of heat transfer for the flow of fluids studied with Brinkman numbers at least as high as 4.22×10^{-3}.

NOTATION

A = area across the flow
$B_0, B_1, B_2, \ldots$ = constants
Br_0 = Brinkman number at entrance, $4\eta_w Q^2/\pi T_0 D^2 k$
$C_1, C_2, C_3, \ldots$ = constants
C_p = heat capacity at constant pressure
D = pipe inside diameter
g = gravitational acceleration
Gr^* = modified Grashof number, $g\rho^2 D^4 q''\beta/k\eta^2$
Gz_x = local Graetz number, $\dot{m}\, C_p/kx$
k = thermal conductivity of fluid
K_b = power law consistency evaluated at bulk temperature
K_w = power law consistency evaluated at the wall temperature
$\dot{m}$ = mass flow rate
$m_1, m_2, m_3, \ldots$ = constants
n = flow behavior index from power law model
Nu_{x0} = local Nusselt number obtained from constant property solution of Sellars, Tribus, and Klein (1956)
Nu_x = local Nusselt number, $q''D/k(T_w - T_b)$
Pr = Prandtl number, $\eta C_p/k$
q'' = heat flux
Q = volume flow rate
r = radial coordinate
R = universal gas constant
Ra^* = modified Rayleigh number, $g\rho^2 D^4 q''\beta C_p/k^2\eta$
Ra_b^* = modified Rayleigh number evaluated at local bulk conditions
Ra_w^* = modified Rayleigh number evaluated at local wall conditions
Ra_{tr}^* = modified Rayleigh number at onset of significant secondary flow
Re = Reynolds number, $\rho VD/\eta$
T_b = local bulk temperature
T_0 = entrance temperature
T_w = local wall temperature
u = velocity in axial direction
V = average velocity in axial direction
x = axial position, measured from entrance to heated boundary
$X_1, X_2, X_3, \ldots$ = independent variables

Greek Letters

β = coefficient of thermal expansion
$\dot{\gamma}_w$ = local wall shear rate $|du/dr|_w$
δ_0 = entrance value of δ
δ_x = local shear rate ratio, $\dot{\gamma}_w/(8V/D)$
ΔH = activation energy of flow
η_b = apparent viscosity, $\tau/\dot{\gamma}$, evaluated at bulk flow conditions
η_w = apparent viscosity, $\tau/\dot{\gamma}$, evaluated at wall conditions
ρ = density
τ = shear stress
τ_w = local shear stress at pipe wall
τ_1 = shear stress at the rotor surface
τ_2 = shear stress at the cup surface
ϕ = dimensionless heat flux, $q''D/T_0 k$
Ω = rotor angular speed

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Trickle-Bed Reactor Performance

Part I. Holdup and Mass Transfer Effects

Liquid holdup and mass transfer rates were measured in a 2.58-cm I.D. tube, packed with glass beads and granular CuO · ZnO catalyst or β-naphthol particles, and operated as a trickle bed. Gas-to-liquid (water) transport coefficients were determined from absorption and desorption experiments with oxygen at 25°C and 1 atm. Liquid-to-particle mass transfer was studied using β-naphthol particles.

Holdup and both mass transfer coefficients were unaffected by gas flow rate but increased with liquid rate. The data were correlated with equations that could be used for predicting mass transfer coefficients at high temperatures and pressures for use in the reaction studies reported in Part II.

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